

DECIMAL REVIEW

A. INTRODUCTION TO THE DECIMAL SYSTEM

The Decimal System is another way of expressing a part of a whole number. A decimal is simply a fraction with a denominator of 10, 100, 1 000 or 10 000 etc. The number of decimal places refers to how many zeros will be in the denominator. Note that the number 5.62 is read as five point six two.

The <u>first</u> decimal place refers to <u>tenths</u>	$2.3 = 2\frac{3}{10}$
The <u>second</u> decimal place refers to <u>hundredths</u>	$2.31 = 2\frac{31}{100}$
The <u>third</u> decimal place refers to <u>thousandths</u>	$2.319 = 2\frac{319}{1000}$

Similarly, six decimal places would be a fraction with a denominator of 1 000 000 (millionths). The most common usage of decimals is in our monetary system where 100 cents (2 decimal places) make up one dollar. For example, \$2.41 is really two dollars and forty-one hundredths ($\frac{41}{100}$) of a dollar.

<u>Examples:</u>	1. Change 2.30 to a fraction Notice that 2.30 is the same as 2.3 In fact, $2.30 = 2.300 = 2.3000$ etc.	$2.30 = 2\frac{30}{100} = 2\frac{3}{10}$
	2. Change 0.791 to a fraction Notice that $0.791 = .791$ The zero in front of the decimal place is not needed.	$0.791 = \frac{791}{1000}$
	3. Change .003 to a fraction Notice that the zeros in this example <u>are</u> important.	$.003 = \frac{3}{1000}$
	4. Simplify 0.0024000 Notice that zero at the end or zero as a whole number (to the left of the decimal) is not needed.	$0.0024000 = .0024$

B. OPERATIONS WITH DECIMALS

1. Addition:

Add \$2.50 and \$1.35

Place numbers in columns, so that the decimal places are in line. Place the decimal point in the same line for the answer.

$$\begin{array}{r} \$2.50 \\ + \$1.35 \\ \hline \$3.85 \end{array}$$

Now, add as if adding whole numbers.

Add 1.3928 and 12.43 and .412

Add zeros to fill in columns. This will not change the value of the number (see examples on previous page.)

$$\begin{array}{r} 1.3928 \\ + 12.4300 \\ + 0.4120 \\ \hline 14.2348 \end{array}$$

2. Subtraction:

Subtract \$1.30 from \$5.45

Set up columns as in addition, and subtract as if subtracting whole numbers.

$$\begin{array}{r} \$5.45 \\ - \$1.30 \\ \hline \$4.15 \end{array}$$

Calculate 5 minus .2982

Fill in columns with zeros, as in addition.

Note the difference in wording in these two examples.

$$\begin{array}{r} 5.0000 \\ - 0.2982 \\ \hline 4.7018 \end{array}$$

3. Multiplication:

Multiply 2.12 by 4.2

At first, ignore the decimals, and multiply as if calculating 212×42 .

Now, add up the decimal places in both numbers and your answer will have that total number of decimal places.

$$\begin{array}{r} 2.12 \text{ (2 places)} \\ \times 4.2 \text{ (1 place)} \\ \hline 424 \\ 8480 \\ \hline 8.904 \text{ (3 places)} \end{array}$$

Product of 0.0941 and .02

Multiply. Add enough zeros to show the correct number of decimal places

$$\begin{array}{r} .0941 \text{ (4 places)} \\ \times .02 \text{ (2 places)} \\ \hline .001882 \text{ (total = 6)} \end{array}$$

Multiply .5624 by 1000

Notice $.5624 \times 1000 = 562.4$, so multiplying $\times 1000$ (0 decimal places) is the same as moving the decimal place 3 places (since 1000 has 3 zeros) to the right.

$$\begin{array}{r} .5624 \quad (4 \text{ places}) \\ \times 1000 \quad (0 \text{ places}) \\ \hline 562.4000 \quad (\text{total} = 4) \end{array}$$

Similarly,

$$\begin{array}{ll} .58 \times 10 = 5.8 & 5.636 \times 10\,000 = 56\,360 \\ .58 \times .1 = .058 & .58 \times .001 = .00058 \end{array}$$

or 562.4

Division: Divide 2.322 by .12

The divisor (.12) must be changed to a whole number (12) by moving the decimal point 2 places to the right, in both numbers. In the answer, place the decimal point directly above the numbers. Note that zeros must be added to complete the division.

$$\begin{array}{r} . \\ .12 \overline{)2.322} \\ \underline{12} \\ 112 \\ \underline{108} \\ 42 \\ \underline{36} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Divide .003 into 51

Firstly, notice the difference in wording in these two division questions. Zero must be added to the number 51 in order to move the decimal 3 places to the right.

$$\begin{array}{r} .003 \overline{)51} \\ \underline{3} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

$$\begin{array}{r} 17000. \\ 3 \overline{)51000.} \\ \underline{3} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

Divide 254.25 by 1000

Notice $254.25 \div 1000 = .25425$. So, dividing by 1000 is the same as moving the decimal 3 places to the left (since 1000 has 3 zeros).

$$\begin{array}{r} 0.25425 \\ 1000 \overline{) 254.250} \\ \underline{200\ 0} \\ 54\ 25 \\ \underline{50\ 00} \\ 4\ 250 \\ \underline{4\ 000} \\ 2500 \\ \underline{2000} \\ 5000 \\ \underline{5000} \\ 0 \end{array}$$

Similarly, $32.52 \div 10 = 3.252$
 $2.6 \div 10\ 000 = .00026$

EXERCISE 1: Decimal Operations Do not use a calculator.

a) Change to fractions: (remember to reduce)

- 1) 5.8 2) 27.3400 3) 30.02 4) 0.590 5) 3.075

b) Addition:

- 1) $2.49 + .32$ 2) 0.042 plus .00982 3) 7.342 and 2 and 7.65
4) $1743.2 + 2.984 + 12.35$ 5) 2.76 more than 8.4590

c) Subtraction:

- 1) 2.036 from 4.478 2) 12.258 from 13 3) 670.1 minus 589.213
4) 0.1002 minus 0.05 5) 19.6 decreased by 5.349

d) Multiplication:

- 1) .21 by .04 2) .42 x .218 3) .75 times 132.786
4) Product of .009 and 2.003 5) .25 of 288
6) 9.4325 by 1000 7) 9.4325 by .001

e) Division:

- 1) 248 divided by 0.8 2) 15.47 divided by .091
3) 40.4 into 828.2 4) 0.0338 divided by 1.30
5) .0025 into 1.875 6) 923.56 divided by 1000
7) 923.56 divided by .01

C. CONVERSION AND ROUNDING OFF

1. Converting Decimals to Fractions:

When changing decimals to fractions, simply create a fraction with 10, 100, 1000 etc. in the denominator. The number of zeros in the denominator is the same as the number of decimal places.

$$0.913 = \frac{913}{1000}$$
$$5.25 = 5\frac{25}{100} = 5\frac{1}{4}$$

2. Converting Fractions to Decimals:

When changing a fraction to a decimal, simply divide the denominator into the numerator. Every time a fraction is changed to a decimal, the division will either stop (as in $\frac{4}{5} = 0.8$) or the division will go on forever

$$\frac{4}{5} = 4 \div 5 = 0.8$$

by repeating (as in $\frac{2}{3} = .\dot{6}$). Notice that a repeating decimal is shown by a dot over the number (if only one number repeats) or as a bar (if more than one number repeats, as in $\frac{2}{11} = .\overline{18}$)

$$\frac{2}{3} = 2 \div 3 = 0.666\dots = \dot{6}$$

$$\frac{2}{11} = 2 \div 11 = .1818\dots = \overline{18}$$

3. Repeating Decimals

Following is a list of some repeating decimals:

$$\frac{1}{3} = .\dot{3} \quad \frac{1}{6} = .1\dot{6} \quad \frac{1}{7} = .\overline{142857} \quad \frac{2}{3} = \dot{6} \quad \frac{5}{6} = .8\dot{3}$$

$$\frac{1}{9} = \overline{.11} \text{ or } .1\dot{1}$$

$$\frac{2}{9} = \overline{.22} \text{ or } .2\dot{2}$$

$$\frac{4}{9} = \overline{.44} \text{ or } .4\dot{4}$$

$$\frac{5}{9} = \overline{.55} \text{ or } .5\dot{5}$$

$$\frac{7}{9} = \overline{.77} \text{ or } .7\dot{7}$$

$$\frac{8}{9} = \overline{.88} \text{ or } .8\dot{8}$$

4. Rounding off:

If we want to divide \$2.00 into 3 equal parts, we would want our answer to be to the nearest cent (or nearest hundredth). Since our answer is closer to 67 cents than 66 cents, we would round off our answer to \$0.67.

$$\begin{array}{r} 0.666\dots \\ 3 \overline{) 2.000} \end{array}$$

When rounding to the nearest thousandth, we want our answer to have 3 decimal places.

$$2.6549 \text{ to nearest thousandth} = 2.655$$

$$2.6549 \text{ to nearest hundredth} = 2.65$$

If the fourth decimal place has a 5 or greater, round up. If less than a 5, do not round up.

$$3.95 \text{ to nearest tenth} = 4.0$$

EXERCISE 2: Conversion and Rounding Off

a) Change to Fractions: (Remember to reduce whenever possible)

1. 2.591 2. 25.030 3. 50.0250 4. 0.8

b) Change to Decimals: (Do not round off)

1. $\frac{7}{8}$ 2. $\frac{3}{11}$ 3. $\frac{4}{9}$ 4. $\frac{3}{5}$ 5. $\frac{4}{7}$
6. $\frac{5}{12}$ 7. $\frac{5}{6}$ 8. $\frac{2}{3}$ 9. $\frac{7}{2}$ 10. $\frac{2}{9}$

c) Round Off:

1. 2.864 to nearest hundredth 2. 35.9649 to nearest thousandth
3. 931.85 to nearest tenth 4. 2.091 to nearest tenth
5. 11.898 to nearest hundredth 6. 12.92 to nearest whole number
7. 11.74235 to nearest thousandth 8. $\frac{5}{9}$ to nearest thousandth
9. $\frac{7}{12}$ to nearest hundredth

ANSWERS

EXERCISE 1: Decimal Operations

- a) 1) $5\frac{4}{5}$ 2) $27\frac{17}{50}$ 3) $30\frac{1}{50}$ 4) $\frac{59}{100}$ 5) $3\frac{3}{40}$
- b) 1) 2.81 2) .05182 3) 16.992 4) 1758.534 5) 11.219
- c) 1) 2.442 2) .742 3) 80.887 4) .0502 5) 14.251
- d) 1) .0084 2) .09156 3) 99.5895 4) .018027 5) 75
- 6) 9432.5 7) .0094325
- e) 1) 310 2) 170 3) 20.5 4) .026 5) 750
- 6) .92356 7) 92356

EXERCISE 2: Conversion and Rounding Off

- a) 1) $2\frac{591}{1000}$ 2) $25\frac{3}{100}$ 3) $50\frac{1}{40}$ 4) $\frac{4}{5}$
- b) 1) .875 2) $\overline{.27}$ 3) $\dot{.}4$ 4) .6 5) $\overline{.571428}$
- 6) $\dot{.}41\dot{6}$ 7) $\dot{.}8\dot{3}$ 8) $\dot{.}6$ 9) 3.5 10) $\dot{.}2$
- c) 1) 2.86 2) 35.965 3) 931.9 4) 2.1 5) 11.90
- 6) 13 7) 11.742 8) .556 9) .58