Transposing Formulae (Electrical and Power Engineering only)

In mathematics, engineering and science, formulae are used to relate physical quantities to each other. They provide rules so that if we know the values of certain quantities, we can calculate the values of others. So, in order to solve equations, formulae can be transposed, or transformed, or rearranged.

### Transposition of Simple Formulae

**Example 1**
Consider the formula \( v = u + at \). Suppose we wish to transpose this formula to obtain a formula for \( t \).
Because we want to obtain \( t \) on its own we start by subtracting \( u \) from each side:
\[
    v = u + at \\
    v - u = at
\]
We now divide everything on both sides by \( a \).
\[
    \frac{v - u}{a} = \frac{at}{a}
\]
\[
    a \div a = 0, \text{ and so finally } t = \frac{v - u}{a}
\]
We have transposed the formula to find an expression for \( t \).

**Example 2**
Consider the formula \( v^2 = u^2 + 2as \) and suppose we wish to transpose it to find \( u \).
We want to obtain \( u \) on its own and so we begin by subtracting \( 2as \) from each side.
\[
    v^2 = u^2 + 2as \\
    v^2 - 2as = u^2
\]
Finally, taking the square root of both sides:
\[
    u = \sqrt{v^2 - 2as}
\]
Notice we need to take the square root of the whole term \( \sqrt{v^2 - 2as} \) in order to find \( u \).

**Example 3**
Consider the formula \( s = ut + \frac{1}{2}at^2 \).
Suppose we want to transpose it to find \( a \).
Because we want \( a \) on its own, we begin by subtracting \( ut \) from both sides.
\[
    s = ut + \frac{1}{2}at^2 \\
    s - ut = \frac{1}{2}at^2
\]
Then we can multiply both sides by 2:
\[
    2(s - ut) = at^2
\]
Example 3 (continued)

And then, to get $a$ on its own, divide both sides by $t^2$:

$$\frac{2(s - ut)}{t^2} = a$$

Now you try:

Exercise 1

Rearrange each of the following formulae to make the quantity shown the subject:

1. $v = u + at$, $u$
2. $v^2 = u^2 + 2as$, $s$
3. $s = vt - \frac{1}{2}at^2$, $a$
4. $p = 2(w + h)$, $h$
5. $A = 2\pi r^2 + 2\pi rh$, $h$
6. $E = \frac{1}{2}mv^2 + mgh$, $v$

Adapted from:

http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-formulae2-tom.pdf June 2004

Answers below:

1. $u = v - at$
2. $s = \frac{v^2 - u^2}{2a}$
3. $a = \frac{2(vt - s)}{t^2}$
4. $h = \frac{1}{2}(p - 2w)$
5. $h = \frac{A - 2\pi r^2}{2\pi r}$
6. $v = \sqrt{\frac{2(E - mgh)}{m}}$