

Ratio and Proportion Study Guide 12

Ratio: A **ratio** is a comparison of the relationship between two quantities or categories of things. For example, a ratio might be used to compare the number of girls to the number of boys who are enrolled in a school.

There are three ways in which a ratio can be expressed. One point to keep in mind is that the numbers that make up the ratio should be expressed according to the way in which the information is stated in the original problem with the number that represents the first category being listed first and the number that represents the second category being listed second. Each of the three methods used for expressing ratios is demonstrated below to show the ratio of girls to boys who are enrolled at Malaspina High School when the ratio is **two** girls for every **three** boys.

Method (1): One way to write a ratio is by using the word **to** in between the two numbers that represent the comparison being made. Thus, the ratio of **girls to boys** who attend Malaspina High School can be expressed as **2 to 3**.

Method (2): A second way in which a ratio can be written is by placing a full colon (**:**) in between the two numbers that represent the comparison being made. Thus, the ratio of **girls to boys** who attend Malaspina High School can be expressed as **2:3**.

Method (3): The most common way to write a ratio is in fraction form. When writing ratios in fraction form, the number that represents the **first category** mentioned in the problem is written as the **numerator** and the number that represents the **second category** is written as the **denominator**.

Thus, the ratio of girls to boys who attend Malaspina High School can be expressed as $\frac{2}{3}$.

Note #1: When two numbers being used to indicate a ratio are written as a fraction, the fraction can be expressed in simplest form in order to make the problem easier to solve by using the greatest possible common factor to reduce the numerator and the denominator of the ratio numbers.

Example: There are **21** boys and **35** girls in a school choir. Therefore, the ratio of boys to girls is $\frac{21}{35}$.

However, both 21 and 35 can be evenly divided by the common factor **7**. As a result, the original ratio expressed as a fraction can be reduced to lower terms:

$$\begin{aligned} \underline{21} \div 7 &= \underline{3} \\ \underline{35} \div 7 &= \underline{5} \end{aligned}$$

Note #2: When the numerator of a ratio that is written in fraction form is larger than the denominator, it will form an improper fraction. When this happens, the fraction should be reduced to lowest terms, but left as an improper fraction such as $\frac{7}{3}$ or $\frac{5}{1}$ instead of converting it to a mixed numeral ($2\frac{1}{3}$) or a whole number (5) as is normally done when expressing an improper fraction in simplest form.

Example: There are 15 horses and 10 donkeys that live on a farm. Express the number of horses as compared to the number of donkeys in ratio form.

Step # 1: Place the number 15 in the numerator position and the number 10 in the denominator position to show the ratio as an improper fraction: $\frac{15}{10}$

Step #2: Check to see if the ratio can be reduced to simpler form. Since both the numerator and the denominator that make-up the ratio can be evenly divided by the common factor **5**, the original fraction can be reduced:

$$\begin{aligned}\frac{15}{10} \div 5 &= \frac{3}{2} \\ 10 \div 5 &= 2\end{aligned}$$

Step #3: Once the original improper fraction has been reduced to simplest form, leave it as an improper fraction: $\frac{3}{2}$

Labeling the Numbers that Make-up a Ratio: When working problems that involve ratios, it is a good idea to label the numbers that make up the ratios in order to avoid making mistakes when solving a problem. The ratio of boys to girls at Malaspina High School used earlier in this study-guide can be used as an example.

Example: There are 2 girls for every 3 boys who attend Malaspina High School. Express the ratio of girls to boys in fraction form and label both numbers that make-up the ratio to indicate what they represent:

$$\begin{array}{l} \text{girls } \frac{2}{3} \\ \text{boys } \end{array}$$

Creating the Ratio Number that Represents the Combined Total of the Categories Being Compared: Some ratio-proportion problems require that a ratio number be used that represents the combined total of the two categories being compared. The ratio number that represents the total can be created by simply adding the two ratio numbers that represent each of the ratio categories.

Example: If the ratio of girls to boys in a class is 2 to 3, what is the **ratio number** that represents the total number of students in the class?

ratio for girls 2

ratio for boys +3

ratio for total students 5

Proportion: A **proportion** is a comparison between two **equal** ratios. Two ratios are said to form an exact proportion if they represent the same fraction quantity. There are two methods that can be used in order to determine whether or not two ratios form an exact proportion.

Method #1: Reducing Each Ratio to Lowest Terms: Write both ratios in fraction form and reduce them to simplest form. If both ratios being compared can be reduced to the same fraction, the original ratios form an exact proportion.

Example: Compare the following two ratios: $\frac{16}{20}$ and $\frac{8}{10}$

Step #1: Analyze the first ratio written as a fraction to determine whether or not it is in simplest form.

Since both 16 and 20 can be evenly divided by the common factor **4**, the original ratio number $\frac{16}{20}$ can be reduced:

$$\begin{aligned}\frac{16}{20} \div 4 &= \frac{4}{5} \\ \frac{20}{20} \div 4 &= 5\end{aligned}$$

Step #2: Analyze the second ratio written as a fraction to determine whether or not it is in simplest form. Since both 8 and 10 can be evenly divided by the common factor **2**, the original ratio number

$\frac{8}{10}$ can be reduced:

$$\begin{aligned}\frac{8}{10} \div 2 &= \frac{4}{5} \\ \frac{10}{10} \div 2 &= 5\end{aligned}$$

Step #3: Once the ratios being compared have been reduced to simplest form, compare them to determine whether or not they are equal. In this example, both ratios can be reduced to $\frac{4}{5}$. Therefore, the two original ratios form an exact proportion.

Method #2: Using the Cross-Multiplication Process: Write both ratios in fraction form and align them side-by-side. Cross-multiply the numerator of the first ratio times the denominator of the second ratio and compare it to the answer you get by cross-multiplying the denominator of the first ratio and the numerator of the second ratio. If the answers you get are the same, the original ratios form an exact proportion.

Step #1: Write the two ratios being compared in fraction form and place them side-by-side

$$\frac{16}{20} \quad \frac{8}{10}$$

Step #2: Cross-multiply the numerator of the first ratio (16) times the denominator of the second ratio (10) and the denominator of the first ratio (20) times the numerator of the second ratio (8).

$$\begin{array}{ccc} 16 & \swarrow & 8 \\ \frac{16}{20} & \times & \frac{8}{10} \\ 20 & \nwarrow & 10 \end{array}$$

$$16 \times 10 = \mathbf{160}$$

$$20 \times 8 = \mathbf{160}$$

Step # 3: Analyze the results of the cross-multiplication process you did in Step # 2. Since the product of both multiplication processes was 160, it proves that the two original ratios are in exact proportion to one another.

Translating Information from Word-Problems into the Correct Ratio-Proportion Relationship: In translating a ratio-proportion word-problem, make sure that the categories that make-up each of the ratios being compared match exactly on both sides of the equal sign. An easy way to make sure you have set-up your ratios correctly is by labeling all of the numbers that make-up the ratios being compared.

Example: The ratio of the number of **girls** as compared with **boys** in a class is 2 to 3. The actual number of girls is 10 and the actual number of boys is 15. Write the two ratios as a proportion.

$$\begin{array}{l} \mathbf{girls} \frac{2}{3} = \mathbf{girls} \frac{10}{15} \\ \mathbf{boys} \end{array}$$

Finding the Unknown Amount When One of the Four Numbers is Missing in a Ratio-

Proportion Problem: By using the cross-multiplication procedure discussed earlier in this study-guide, it is possible to determine the missing number in a ratio-proportion problem. In performing such operations, a letter such as **N** can be used to substitute for the missing number you are trying to find. In the process of solving the problem, one ratio number will be multiplied times the unknown amount (N) and placed directly next to the letter N on one side of the equal sign (=) in an equation, and the product you get by multiplying the other two numbers will be placed on the opposite side of the equal sign. After the multiplication process has been completed, divide the number connected to the letter N into the number on the opposite side of the equal sign in order to find the missing number.

Example: If the ratio of horses to cows living on a farm is 5 to 3, how many horses are there if there are 24 cows?

Step #1: Write the ratio of horses to cows on one side of an equal sign and label the numbers that make-up the ratio:

$$\begin{array}{l} \text{horses } \underline{5} = \\ \text{cows } 3 = \end{array}$$

Step #2: Write the actual number of horses and cows as a ratio on the opposite side of the equal sign. Use the letter (N) to represent the missing quantity (the number of horses). Remember to match the categories of the ratio numbers and the actual number of the things being compared in exactly the same way on each side of the equal sign. Use labels to be sure you are placing the numbers in their proper positions:

$$\begin{array}{l} \text{horses } \underline{5} = \underline{N} \text{ horses} \\ \text{cows } 3 = 24 \text{ cows} \end{array}$$

Step # 3: Check to see if the fraction that represents the ratio of horses to cows can be reduced. Since there are no other common factors for **5** and **3** other than the number **1**, the ratio cannot be reduced.

Step #4: Cross-multiply the numbers on either side of the equation (remember that one of the two numbers you will get by multiplying will end up next to the letter N):

$$\begin{array}{l} \text{horses } \underline{5} \quad \swarrow \quad \searrow \quad \underline{N} \text{ horses} \\ \text{cows } 3 \quad \nwarrow \quad \nearrow \quad 24 \text{ cows} \end{array}$$

$$5 \times 24 = 3 \times N$$

$$120 = 3N$$

Step #4: Once the cross-multiplication process has been completed, divide the number connected to the letter N (3) into the number on the opposite side of the equal sign (120):

$$3 \overline{)120} \begin{array}{r} 40 \\ \end{array}$$

Note: The result of the final computation shows that if the ratio of horses to cows living on a farm is 5 to 3, and the actual number of cows is 24, then the actual number of horses is 40.

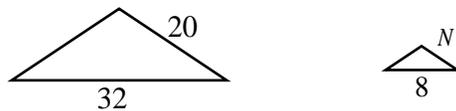
Using the Ratio- Proportion Formula to Solve Problems Involving Similar Geometric Figures:

The ratio-proportion formula can be used to determine the length of the side of one geometric figure that is missing when you know the length of the same side (corresponding side) of another similar figure.

Similar Figures: Geometric figures are considered to be similar when they have the same shape and when the corresponding angles formed where their sides meet measure the same number of degrees. It should be recognized that even though the angles measure the same number of degrees, the sides of similar geometric figures are different in length.

When solving problems in which you are trying to determine the length of the missing side of one of two similar geometric figures, you should form a ratio using two sides of one of the figures and compare it to the ratio of the same sides of the other figure. When doing this, the numbers representing the lengths of the sides that are similar should be placed in the same positions in both ratios. Use a letter such as N or X to represent the side of the geometric figure that is missing.

Example #1: The two triangles in this problem are similar. Determine the length of the side of the triangle that is missing represented by the letter N.



Step #1: Form a ratio using the two sides that are identified in the first triangle: $\frac{20}{32}$

Step #2: Reduce the ratio if possible in order to make the problem easier to solve. Since **4** is a common factor for both **20** and **32**, the original ratio number can be reduced:

$$\begin{aligned} \underline{20} \div 4 &= \underline{5} \\ 32 \div 4 &= 8 \end{aligned}$$

Step #3: Form a ratio using the two sides of the second figure that correspond with the sides of the first figure by placing the number and letter representing the sides of the second figure in the same positions as they are in the first ratio. The side that is missing is represented by the letter **N**:

Step #4: Position the two ratios on either side of an equal sign (=): $\frac{5}{8} = \frac{N}{8}$

Step #5: Use the cross multiplication process discussed earlier in this study-guide:

$$\begin{array}{ccc} \underline{5} & \swarrow & \searrow \underline{N} \\ 8 & \nearrow & \nwarrow 8 \end{array}$$

$$8 \times N = 5 \times 8$$

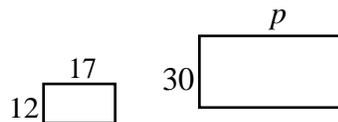
$$8N = 40$$

Step #6: Divide the number 8 connected to the letter N into the number on the opposite side of the equal sign (40):

$$8 \overline{)40} \begin{array}{r} 5 \\ \end{array}$$

Note: The length of the side that is missing is **5**. You might have noticed that once the original ratio 20/32 was reduced to simplest form, it had the same denominator (8) as the ratio to which it was being compared. Therefore, the numerators of both ratios ended up being the same as well (5).

Example #2: Determine the length of the side that is missing from the largest of the two similar rectangles shown to the right:



Step #1: Form a ratio using the two sides that are identified in the first rectangle: $\frac{12}{17}$

Step #2: Reduce the ratio if possible in order to make the problem easier to solve. Since there are no common factors for 12 and 17 other than the number 1, the ratio cannot be reduced and therefore is already in simplest form.

Step #3: Form a ratio using the lengths of the two sides of the larger rectangle by placing the sides that correspond with the smaller triangle in the same positions in the second ratio. The side of the larger triangle that is missing is represented by the letter **P**: $\frac{30}{P}$

Step #4: Position the two ratios on either side of an equal sign: $\frac{12}{17} = \frac{30}{P}$

Step #5: Perform the cross-multiplication process:

$$\begin{array}{ccc} \frac{12}{17} & \begin{array}{c} \nearrow \\ \searrow \end{array} & \frac{30}{P} \end{array}$$

$$17 \times 30 = 12 \times P$$

$$510 = 12P$$

Step #6: Divide the number 12 connected to the letter **P** into the number on the opposite side of the equal sign (510).

$$\begin{array}{r} 42.5 \\ 12 \overline{)510} \end{array}$$

Note: The result of the computation shows that the length of the missing side of the larger of the two similar rectangles being compared is **42.5**.

Practice:

http://www.mathx.net/downloads/proportions/MATHX.NET-Checking_proportion-medium.pdf

http://www.mathx.net/downloads/proportions/MATHX.NET-Similar_figures-medium.pdf